MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

EEM2036 – ENGINEERING MATHEMATICS III

(All Sections/Groups)

5 OCTOBER 2015

2.30 PM – 4.30 PM

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of **6 printed pages** (including cover page and formula sheets) with **four questions** only.
2. Attempt **ALL questions.** All questions carry equal marks and the distribution of marks for each question is given.
3. Please write all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

a) The position (x_1, x_2) of an object on the x_1x_2-plane is determined by the following system of differential equations

$$\frac{dx_1}{dt} = -6x_1 + 2x_2$$

$$\frac{dx_2}{dt} = -3x_1 + x_2$$

given that the coefficient matrix $\begin{pmatrix} -6 & 2 \\ -3 & 1 \end{pmatrix}$ of the above system has eigenvectors $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ corresponding to eigenvalues 0 and -5 respectively. Find the general solution of x_1 and x_2. [12 marks]

b) Using the change of variables, $u = 2xy$ and $v = x^2 - y^2$, evaluate

$$\iint_D (x^2 + y^2)dxdy$$

where D is the region in the first quadrant bounded by $2xy = 2$, $2xy = 4$, $x^2 - y^2 = 1$ and $x^2 - y^2 = 2$. [13 marks]

Question 2

a) Consider the following data:

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

Construct the Newton's divided difference interpolating polynomial to approximate $f(9)$. [15 marks]

b) Find the root of $x^4 - x = 10$ correct to 3 decimal places by taking initial value as $x_0 = 2$. Solve using Newton- Raphson method. [10 marks]

Continued...

Question 3

a) i) Determine whether the vector field

$$\vec{F} = \left(y\cos(xy)\right)\hat{i} + \left(x\cos(xy)\right)\hat{j} - \sin z\hat{k}$$

is conservative. If it is, find a function f such that $\vec{F} = \nabla f$. [9 marks]

ii) A particle moves by following the path xy^2 from $(1, 0.2, 0.1)$ to $(0, 1.4, 0.4)$ under the influence of the field $\vec{F}$. Find the potential between those two points. [4 marks]

b) Use the Divergence theorem to calculate the flux of the force $\vec{F} = x^3\hat{i} + x^2y\hat{j} + x^2z\hat{k}$ across the surface of a cylinder $x^2 + y^2 = 9$ and the circular disks of $z = 0$ and $z = 4$. [12 marks]

Question 4

a) Solve the initial value problem at $x = 0.2$ given $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ by using:

i) Runge Kutta method of order two. [9 marks]

ii) Euler's method. [3 marks]

b) Suppose the temperature at a point in a metal plate is given by

$T = 80 - 20xe^{-\frac{1}{20}\left(x^2+y^2\right)}$ where the center of the plate is taken to be at (0,0).

i) At the origin, in what direction the temperature would increase and decrease most rapidly? [6 marks]

ii) At the origin but in the direction of the unit vector $\dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j})$, what is the rate of change? [2 marks]

c) In rectangular coordinates, determine whether the vector field $\vec{F}$ is solenoidal or irrotational. $\vec{F} = 2x^2y\hat{i} - xyz^3\hat{j} + 3xz^2\hat{k}$ [5 marks]

Continued...

APPENDIX

TABLE OF FORMULAS

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \ldots, x_k](x - x_0)\cdots(x - x_{k-1})$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0)$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n)$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

with the error term

$$\left|\frac{h^2}{6} f^{(3)}(c_x)\right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}[f(a) + f(b)]$$

for some ξ in (a, b) and $h = b - a$, with the error term is $\left|\frac{h^3 f''(\xi)}{12}\right|$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{j=1}^{n-1} f(x_j)\right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term is $\left|\frac{(b-a)h^2 f''(\xi)}{12}\right|$.

10. Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

for some ξ in (a, b) and $h = \frac{b-a}{2}$, with the error term $\left|\frac{h^5}{90} f^{(4)}(\xi)\right|$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a) + 2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j}) + 4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1}) + f(b)\right]$$

for some ξ in (a, b) and $h = \frac{b-a}{n}$, with the error term $\left|\frac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$

Continued ...

12. Newton-Raphson's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \qquad n = 0,1,2,\ldots$$

13. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2} Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

14. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

15. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1\right),$$
$$k_3 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2\right),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

End of Paper